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LETTER TO THE EDITOR

Note on a suggested formation of charge from an electromagnetic wave

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Abstract. Jennison has described an historic experiment in which he succeeds in bringing an electromagnetic wave to rest in the laboratory. It is here pointed out that the electrostatic field could not be produced by static charge in the laboratory; the concept of the formation of such charge from the original wave therefore appears to be somewhat misdirected.

Questions arising from Jennison's experiment

Professor R C Jennison has kindly shown me the remarkable and ingenious experiment which he has recently described in this journal (Jennison 1982). Also he has called my attention to the interesting questions of physical principle to which it gives rise, as indicated in his paper. This note is an attempt in principle to answer three questions, which may be formulated as follows.

(a) The experiment produces the equivalent of monochromatic electromagnetic radiation in a dielectric. The wave motion is brought to rest in the laboratory. All that remains observable is ostensibly an electrostatic field. Were this in fact the case, it would be natural to infer that this field could be reproduced by some particular distribution of electric charge also at rest in the laboratory. In the phraseology of the title of Jennison's paper, do we have thus a phenomenon of 'the formation of charge (in fact, the particular distribution just mentioned) from a travelling electromagnetic wave by reduction of the effective velocity of light to zero'? Or can it be argued, as stated in his text, 'that the field system forms a charge where no charge existed before'?

(b) In the original travelling wave there is both an electric field and a magnetic field. When the electric field has been rendered apparently static, is there still a non-zero magnetic field?

(c) What Jennison seeks to reproduce is a laboratory moving with an electromagnetic wave which is travelling through a dielectric with velocity $\boldsymbol{V}$ relative to the dielectric. So he must cause his 'dielectric' to move with velocity $-\boldsymbol{V}$ relative to his laboratory. In order to achieve this without the dielectric flying out of the laboratory window, Jennison causes the wave to move round in a circular transmission line, and then causes this line of 'dielectric' to rotate with equal and opposite speed. Such circular motion is the only available means for maintaining steady motion within a laboratory. But does the rotation as such play any essential part in the outcome of the experiment?

Qualitative treatment

Question (c) enters Jennison's published paper only by implication; but he raised it explicitly in conversation with the writer. The answer is that the rotation as rotation plays no essential part. We see this by discussing first the case of uniform linear motion. Having discussed the physics in this case we shall see that, if instead of such motion we have to do with steady rotation, this can make no significant change in that physics. For the moment we therefore restrict ourselves to inertial frames of reference.

Question (a) may be treated in several ways; *all show that there is no formation of charge.* The most general answer is that we are dealing with a given electromagnetic field, including its sources, as described first with reference to some one inertial frame. We are then concerned with the description of the *same field* using another inertial frame. But the field is in an absolute sense the same entity. If we make the common distinction betweeen the 'field' and the 'sources' of the field, then either may be said to determine the other by means of Maxwell's equations. In this sense, Jennison is of course correct about the 'chicken-and-egg' concept. However, it is always a case of 'same chicken, same egg' that is to say, if two observers in relative motion are in the same electromagnetic field and both observe it using the same standard procedure, they obtain different descriptions; if they infer the sources of the field, again they arrive at different descriptions. But in the everyday physical sense, they are descriptions of the same physical system—the same dynamo, the same set of charges, or whatever. All this is expressed most simply by saying that any given electromagnetic system is described by its field *tensors* and associated source *vector* in space–time that are the *same* tensors and vector for all observers. Different observers simply find it natural to employ different frames of reference. A change of frame cannot produce electric charge where none existed before.

Applying this to Jennison's static field, its only possible sources are the actual sources of the original travelling wave. The observed static field and the original wave are the same absolute entity (or parts of it) with the same source or sources.

A slightly different viewpoint is that if the static field could be produced by some distribution of charge that is static in the same frame, then this *is* the source of this field. And if the static field is derived from a travelling wave then the same charge would be the source of this wave. But in the frame in which the wave is travelling the charge would have purely uniform motion and this could not generate a travelling wave. So once again we see that the static field could not be produced by static charge in its frame.

The simplest answer in Jennison's particular case is that if an observer is moving with the wave produced by some given source, *he is forever under the influence of the source as it was at one particular instant in the source's proper time*, namely the instant when the element of the wave at the observer left the source. In such a case, the field at the observer is static simply because it is always due to one and the same state of the source. If *the source is purely electrical the field is produced by this electrical system in one configuration, and so it is electrostatic*; *if the source is purely magnetic, the field at the observer is magnetostatic*; if the source is a combination, the field is also a combination. Thus we have here also answered question (b).

A rough way of expressing this last answer is to say that the observer moving with the wave always 'sees' the source precisely as it was when the element of the wave at himself—which is always the same element since he is moving with it—left the source. In the same frame, another observer at a distance l further from the source 'sees' the source as it was at a time earlier by the amount l/V, where V is the speed of the original travelling wave and where we neglect relativistic effects in c^{-2}. So the field at the second position is due always to one state of the source, but not in general the same state as that producing the field at the first position. Thus the *apparently static field in the laboratory frame could not be produced by a static distribution of charge in this frame.* This is another way of reaching the same conclusion as before.

The conclusion is valid but the way we have expressed the argument is not strictly correct. For relative to any observer moving with the wave there is zero radiation flux, so that literally he sees nothing! But we allow him the ability to sense the field and we can regard this as a sort of 'seeing'. This is worth mentioning because Jennison stresses that his effect is not 'a stroboscopic artefact.' This is true, of course, as regards his static field itself, but we can interpret that field as a sort of frozen picture of its source.

Analytical treatment

In discussing Jennison's phenomenon we are dealing with 'laboratory' speeds, and so relativistic effects, i.e. terms in c^{-2}, may be neglected throughout. In that case, if S, $\bar{S}$ are inertial frames with rectangular coordinates x, y, z and $\bar{x}$, $\bar{y}$, $\bar{z}$ with the axes of x, $\bar{x}$ along each other, and with $\bar{S}$ moving relative to S with speed V in the x direction, for any event we have

$$\bar{z} = x - Vt, \qquad \bar{y} = y, \qquad \bar{z} = z, \tag{1}$$

where time t is the same in both frames and the origins of coordinates coincide at $t = 0$. If an electromagnetic field as described by observers at rest in S, $\bar{S}$, has at one and the same event the vectors $\boldsymbol{D}$, $\boldsymbol{H}$ and $\bar{\boldsymbol{D}}$, $\bar{\boldsymbol{H}}$ in standard notation, these are related according to

$$\begin{aligned} \bar{D}_x &= D_x, & \bar{D}_y &= D_y - VH_z/c, & \bar{D}_z &= D_z + VH_y/c, \\ \bar{H}_x &= H_x, & \bar{H}_y &= H_y + VD_z/c, & \bar{H}_z &= H_z - VD_y/c. \end{aligned} \tag{2}$$

Now let space be filled with a uniform isotropic dielectric at rest in S having permittivity ε and (for simplicity) permeability $\mu = 1$; with Jennison's case in mind, suppose $\varepsilon \gg 1$.

Electromagnetic wave-speed in S is then

$$V = c\varepsilon^{-1/2} \ll c. \tag{3}$$

(So we may not try to check the following results as they stand by considering their limits as $\varepsilon \to 1$, $V \to c$.) Suppose then that the field $\boldsymbol{D}$, $\boldsymbol{H}$ represents a plane-polarised plane wave propagating in the x direction and having $\boldsymbol{D}$ in the y direction. We have the standard properties

$$D_x = 0, \qquad D_z = 0, \qquad H_x = 0, \qquad H_y = 0, \qquad H_z = \varepsilon^{-1/2} D_y. \tag{4}$$

We take the wave to be monochromatic so that we may write

$$D_y = A \sin[n(x - Vt)] \tag{5}$$

where A, n are constants.

The case of interest for the present purpose is that in which *the frame $\bar{S}$ moves with the wave* so that in (1), (2), V is in fact the wave speed (3). Then (1), (2), (4),

(5) yield

$$\bar{H}_z = \bar{H}_y = 0 \qquad \text{and} \qquad \bar{H}_z = (\varepsilon^{-1/2} - V/c)D_y = 0;$$

$$\bar{D}_x = \bar{D}_z = 0 \qquad \text{and} \qquad \bar{D}_y = (1-\varepsilon^{-1})D_y \approx A \sin n\bar{x}. \qquad (6)$$

Thus an observer moving with the wave observes a static electric field and a zero magnetic field. This reproduces in the simplest possible way these particular features of Jennison's experiment.

This does not, however, deal with the problem of the source or sources of the field in either S or $\bar{S}$. As already said, we are dealing with a given field and contemplating simply two possible descriptions thereof. As regards the charge-current four-vector in the plane wave, this vanishes everywhere in the S description, so it vanishes in the $\bar{S}$ description. That is to say, the field has no source in either description. If it could be set up, it would satisfy Maxwell's equations everywhere, but it never could be set up.

The simplest practical realisation of a plane wave is the field in a region at a large distance x, say, from an electric dipole oscillator. This is like the field considered above except that the amplitude A is no longer constant but takes the form

$$A = a/x$$

where a is a constant along a given direction of propagation. We may take this x to be the same as in (1). Corresponding to (6) we then have

$$\bar{D}_y = a(Vt+\bar{x})^{-1} \sin n\bar{x}.$$

In this realisable case the field in $\bar{S}$ is thus not actually static; it grows secularly weaker with time.

Using the same rough explanation as before, in this case an observer moving with the wave again 'sees' the source forever in the same configuration, but now 'sees' it receding from himself. In fact, if the observer in $\bar{S}$ observes his field completely, he is bound to infer that it arises from an oscillating dipole receding from himself with the speed V. In practice, we would of course suspect something of the sort since he would be painfully aware of the fact that he is rushing through a dielectric at this speed. Also it is evident that in most practical examples of waves in dielectrics, if an observer contrives to move so that he finds the field near himself to be approximately static, if in the same frame he explores the field more extensively he must find it to be mostly very non-static.

Retuning to Jennison's ingenious example, it seems evident that all the essential features are accounted for by the foregoing physical principles and that no others are required. Thus there is nothing essential that is left over to be ascribed to any effect of rotation; we must conclude that rotation plays no primary role. It does of course enable Jennison to achieve the desired sort of relative motion without secular changes in relative distances so that its main function is to make the steady state more obvious. Also Jennison is able to concentrate his field into the region in which he can render it closely static. Nevertheless it must remain true that if the laboratory physicist could make a sufficiently complete survey of the field in his own frame and could apply Maxwell's equations in order to infer the sources, what he would be bound to infer would be the contrivances that Jennison actually employs in order to 'energise' his system—not any distribution of static charge that could be said to be 'formed' from the field.

I am grateful to Professor Jennison for discussing his experiment with me. Nothing I here venture to write about its interpretation should take anything away from the distinction of this laboratory achievement.

Reference

Jennison R C 1982 *J. Phys. A: Math. Gen.* **15** 405–8